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# GMAT 数学

## 排列组合专题

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词汇：

combination                      组合

permutation                      排列

probability, possibility        概率

注意：排列组合仅仅是解决概率的工具之一，遇到概率题先考虑用简单概率能不能解决，不行的话再考虑排列组合。

基本概念：

**(1) 可能性和概率**

可能性：符合条件的事情的发生共有多少种可能。（正整数）

概率：符合条件的事情的可能性/没有限定条件的情况下总的可能性

**(2) 组合和排列**

组合：从  $m$  个元素（总体数量）中，任取  $n$  个元素（目标数量）组成一组。用  $C$ （combination）表示。

核心是抽取！

$$C_m^n = \frac{m!}{n!(m-n)!}$$

排列：从  $m$  个元素（总体数量）中，任取  $n$  个元素（目标数量），并排成一排。用  $P$ （permutation）表示。

核心是排序！

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$$P_m^n = \frac{m!}{(m-n)!}$$

注意：排列、组合无法解决限定条件、重复元素、按次序讨论等问题。

### (3) 加法原理和乘法原理

**加法原理：**做一件事，完成它可以有  $n$  类办法，在第一类办法中有  $m_1$  种不同的方法，在第二类办法中有  $m_2$  种不同的方法，……，在第  $n$  类办法中有  $m_n$  种不同的方法，那么完成这件事共有  $N=m_1+m_2+m_3+\cdots+m_n$  种不同方法。

**乘法原理：**做一件事，完成它需要分成  $n$  个步骤，做第一步有  $m_1$  种不同的方法，做第二步有  $m_2$  种不同的方法，……，做第  $n$  步有  $m_n$  种不同的方法，那么完成这件事共有  $N=m_1 \times m_2 \times m_3 \times \cdots \times m_n$  种不同的方法。

**区分两个原理：**要做一件事，完成它若是有  $n$  类办法，是分类问题，第一类中的方法都是独立的，因此使用加法原理；做一件事，需要分  $n$  个步骤，步与步之间是连续的，只有将分成的若干个互相联系的步骤，依次相继完成，这件事才算完成，因此用乘法原理。

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### 考查形式 1: 直接代公式

知识点: 分清是抽取 (组合) 还是排序 (排列)。然后直接将数据代入公式。

例题 1:

There are 10 children in a company's day-care center, and a pair of children is to be selected to play a game. At most, how many different pairs are possible?

- (A) 100      (B) 90      (C) 50      (D) 45      (E) 25

解析:  $C_{10}^2$

答案: D

例题 2:

Jill, who lives in City C, plans to visit 3 different cities, M, L, and S. She plans to visit each city exactly once and return to City C after the 3 visits. She can visit the cities in any order. In how many different orders can she visit the 3 cities?

- (A) 3      (B) 6      (C) 8      (D) 9      (E) 12

解析:  $P_3^3$

答案: B

例题 3 (GWD 第 16 套 Q2)

If each of the 12 teams participating in a certain tournament plays exactly one game with each of the other teams, how many games will be played?

- (A) 144      (B) 132      (C) 66      (D) 33      (E) 23

解析:  $C_{12}^2 = 66$

答案: C

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例题 4:

John has 5 friends who want to ride in his new car that can accommodate only 3 passengers at a time. How many different combinations of 3 passengers can be formed from the 5 friends?

- A. 3
- B. 8
- C. 10
- D. 15
- E. 20

解析:  $C_5^3 = 10$

答案: C

例题 5:

一个圆上均匀分布了 10 个点, 从这十个点中取六个点, 可以组成多少个六边形?

思路:

$$C_{10}^6 = 10!/(6!4!) = 210$$

例题 6 (GWD T4Q5)

A certain club has 20 members. What is the ratio of the number of 5-member committees that can be formed from the members of the club to the number of 4-member committees that can be formed from the members of the club?

- (A) 16 to 1      (B) 15 to 1      (C) 16 to 5      (D) 15 to 6      (E) 5 to 4

解析:  $C_{20}^5 : C_{20}^4 = 16 : 5$

答案: C

例题 7 (GWD T9Q7)

A company has assigned a distinct 3-digit code number to each of its 330 employees.

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Each code number was formed from the digits: 2, 3, 4, 5, 6, 7, 8, 9 and no digit appears more than once in any one code number. How many unassigned code numbers are there?

- (A) 6      (B) 58      (C) 174      (D) 182      (E) 399

解析:  $P_8^3 = 330$

答案: A

例题 8 (GWD T10Q6)

If a code word is defined to be a sequence of different letters chosen from the 10 letters *A, B, C, D, E, F, G, H, I, and J*, what is the ratio of the number of 5-letter code words to the number of 4-letter code words?

- (A) 5 to 4      (B) 3 to 2      (C) 2 to 1      (D) 5 to 1      (E) 6 to 1

解析:  $P_{10}^5 : P_{10}^4 = 6 : 1$

答案: E

例题 9 (GWD T13Q3)

Each participant in a certain study was assigned a sequence of 3 different letters from the set  $\{A, B, C, D, E, F, G, H\}$ . If no sequence was assigned to more than one participant and if 36 of the possible sequences were not assigned, what was the number of participants in the study? (Note, for example, that the sequence *A, B, C* is different from the sequence *C, B, A*.)

- (A) 20      (B) 92      (C) 300      (D) 372      (E) 476

解析:  $P_8^3 = 36$

答案: C

例题 10:

有 0.1, 0.001, 0.0001, 0.000001, 0.00000001 五个数值, If two or more numbers added together, how many different numbers we can get?

**思路:**

总共五个数，因为两个数相加，不管谁在前谁在后，和是一样的，所以是组合问题。

两个数相加的可能性:  $C_5^2 = 10$

三个数相加的可能性:  $C_5^3 = 10$

四个数相加的可能性:  $C_5^4 = 5$

五个数相加的可能性:  $C_5^5 = 1$

总共的可能性:  $10+10+5+1 = 26$

**答案:** 26

**考查形式 2: 分组抽选**

一堆数据中，有一些分组，要求每个分组抽取特定的数量然后构成一个整体。

做法: 各组分别抽取，然后相乘。

**例题 1:**

A certain company employs 6 senior officers and 4 junior officers. If a committee is to be created that is made up of 3 senior officers and 1 junior officer, how many different committees are possible?

- (A) 8      (B) 24      (C) 58      (D) 80      (E) 210

解析:  $C_6^3 * C_4^1 = 20 * 4 = 80$

答案: D

**例题 2 (GWD T12Q7)**

There are 11 women and 9 men in a certain club. If the club is to select a committee of 2 women and 2 men, how many different such committees are possible?

- (A) 120      (B) 720      (C) 1,060      (D) 1,520      (E) 1,980

解析:  $C_{11}^2 * C_9^2 = 55 * 36 = 1980$

答案: E

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例题 3:

The buyer of a new home must choose 2 of 4 types of flooring and 2 of 6 paint colors. How many different combinations of type of flooring and paint color are available to the home buyer?

- A. 4
- B. 24
- C. 48
- D. 90
- E. 96

解析:  $C_4^2 * C_6^2 = 6 * 15 = 90$

答案: D

例题 4 (GWD T3Q30):

The membership of a committee consists of 3 English teachers, 4 Mathematics teachers, and 2 Social Studies teachers. If 2 committee members are to be selected at random to write the committee's report, what is the probability that the two members selected will both be English teachers?

- (A) 2/3      (B) 1/3      (C) 2/9      (D) 1/12      (E) 1/24

解析: 总的可能性:  $C_9^2 = 36$ ,

符合条件的可能性:  $C_3^2 = 3$

概率是:  $3/36 = 1/12$

答案: D

例题 5 (GWD T6Q24)

A certain restaurant offers 6 kinds of cheese and 2 kinds of fruit for its dessert platter. If each dessert platter contains an equal number of kinds of cheese and kinds of fruit, how many different dessert platters could the restaurant offer?



- (A) 8      (B) 12      (C) 15      (D) 21      (E) 27

解析:

题目中要求相同数量的 cheese 和 fruit, 有两种可能:

(1) 1 种 cheese 和 1 种 fruit

$$C_6^1 \text{ (6 种 cheese 中抽出 1 种)} * C_2^1 \text{ (2 种 cheese 中抽出 1 种)} = 12$$

(2) 2 种 cheese 和 2 种 fruit

$$C_6^2 \text{ (6 种 cheese 中抽出 2 种)} * C_2^2 \text{ (2 种 cheese 中抽出 2 种)} = 15$$

综上, 共有  $12 + 15 = 27$  种可能。

答案: E

例题 6 (GWD T7Q5)

From a group of 3 boys and 3 girls, 4 children are to be randomly selected. What is the probability that equal numbers of boys and girls will be selected?

- (A)  $\frac{1}{10}$       (B)  $\frac{4}{9}$       (C)  $\frac{1}{2}$       (D)  $\frac{3}{5}$       (E)  $\frac{2}{3}$

解析: 6 个人里抽 4 个人的可能性:  $C_6^4 = 15$

抽到 2 男 2 女的可能性:  $C_3^2 * C_3^2 = 3 * 3 = 9$

概率:  $9/15 = 3/5$

答案: D

例题 7 (GWD T6Q7):

A certain university will select 1 of 7 candidates eligible to fill a position in the mathematics department and 2 of 10 candidates eligible to fill 2 identical positions in the computer science department. If none of the candidates is eligible for a position in both departments, how many different sets of 3 candidates are there to fill the 3 positions?

- (A) 42      (B) 70      (C) 140      (D) 165      (E) 315

解析:  $C_7^1 * C_{10}^2 = 7 * 45 = 315$

答案: E

例题 8 (GWD T16Q27):

A box contains exactly 24 balls, of which 12 are red and 12 are blue. If two balls are to be picked from this box at random and without replacement, what is the probability that both balls will be red?

- (A)  $\frac{11}{46}$       (B)  $\frac{1}{4}$       (C)  $\frac{5}{12}$       (D)  $\frac{17}{40}$       (E)  $\frac{19}{40}$

解析: 总的可能性:  $C_{24}^2$

抽到 2 个红球的可能性:  $C_{12}^2$

答案: A

例题 9:

An analyst will recommend a combination of 3 industrial stocks, 2 transportation stocks, and 2 utility stocks. If the analyst can choose from 5 industrial stocks, 4 transportation stocks, and 3 utility stocks, how many different combinations of 7 stocks are possible?

- A. 12  
B. 19  
C. 60  
D. 180  
E. 720

解析:  $C_5^3 * C_4^2 * C_3^2$

答案: D

例题 10:

In a box of 12 pens, a total of 3 are defective. If a customer buys 2 pens selected at random from the box, what is the probability that neither pen will be defective?

- A.  $1/6$   
B.  $2/9$   
C.  $6/11$   
D.  $9/16$   
E.  $3/4$

解析: 12 支钢笔: 3 个坏的, 9 个没坏。

从 12 支钢笔中抽 2 支钢笔总的可能性:  $C_{12}^2 = 66$

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抽的 2 支钢笔都不是坏的可能性:  $C_9^2 = 36$

概率:  $36/66 = 6/11$

答案: C

例题 11 (11 月机经):

一个部门里共有 10 个女的、6 个男的, 从 16 个人中选两个人参加公司工作会议, 问刚好选到一男一女的概率是多少?

思路:

$$\frac{C_{10}^1 * C_6^1}{C_{16}^2} = \frac{10 * 6}{120} = \frac{1}{2}$$

答案:  $\frac{1}{2}$

例题 12:

有四根电线, 其中两根连在 power source 上, 问选两根电线, 刚好是两根都在 power source 上的几率是多大?

思路:

总共四根电线: 两根电线有 power source, 另外两根电线没有 power source。

4 根电线选 2 根的可能性:  $C_4^2 = 6$

选的 2 根电线都是 power source 的可能性:  $C_2^2 = 1$

所以概率是  $\frac{1}{6}$

答案:  $\frac{1}{6}$

例题 13 (3 月机经):

有两根平行直线, 其中一根有 5 个点, 另一根有 4 个点。问以这些点作顶点 (vertice), 能做出多少个三角形?

思路:

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$$C_5^2 * C_4^1 + C_4^2 * C_5^1 = 70$$

答案：70

### 考查形式 3：依次讨论

特点：数位问题；强调抽选的顺序性，每次抽选有一定的限定条件。

做法：讨论每一个数位或者每一个次序的可能性，然后用乘法原理。

原则：特殊元素，优先处理；特殊位置，优先考虑。

例题 1：（5 月机经）

begin 这个单词中选字母重新排列，以元音开头的情况有多少种？

解析： $2 * 4 * 3 * 2 * 1 = 48$  或  $2 * P_4^4 = 48$

答案：48

例题 2（11 月机经）：

某地的车牌有 4 个 characters，第一个是字母 E，第二个是 26 个字母中任意一个，第三、四个是 0-9 中任意一个数字，皆可重复，求所有可能的车牌号的数量？

思路：

第一个已定 第二个数字有 26 种可能 第三个和第四个各有 10 种可能 所以总数应该是  $26 \times 10 \times 10$

答案：2600

例题 3：

A certain stock exchange designates each stock with a one-, two-, or three-letter

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code, where each letter is selected from the 26 letters of the alphabet. If the letters may be repeated and if the same letters used in a different order constitute a different code, how many different stocks is it possible to uniquely designate with these codes?

- (A) 2,951
- (B) 8,125
- (C) 15,600
- (D) 16,302
- (E) 18,278

解析: (1) 一个字母: 26  
(2) 两个字母:  $26 \times 26$   
(3) 三个字母:  $26 \times 26 \times 26$   
把上面三种情况加起来

答案: E

例题 4:

A company plans to assign identification numbers to its employees. Each number is to consist of four different digits from 0 to 9, inclusive, except that the first digit cannot be 0. How many different identification numbers are possible?

- (A) 3,024
- (B) 4,536
- (C) 5,040
- (D) 9,000
- (E) 10,000

解析:  $9 \times 9 \times 8 \times 7 = 4536$

答案: B

例题 5 (3 月机经):

一个 more than 2000 的 even 四位数, 四个数都是从 {1, 2, 3, 4} 里选的, 不能重复, 问能有多少个这样的数?

思路:

第四位有 2 种选择 (只能是 2 或 4);

第一位不能是 1, 也不能和最后一位重复, 有 2 种选择;

第二位不能和第一位、第四位重复, 有 2 种选择;

第三位不能和第一位、第二位、第四位重复, 只有 1 种选择。

所以共有  $2 \times 2 \times 1 \times 2 = 8$  种

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答案：8

例题 6 (GWD T1Q2):

A certain roller coaster has 3 cars, and a passenger is equally likely to ride in any 1 of the 3 cars each time that passenger rides the roller coaster. If a certain passenger is to ride the roller coaster 3 times, what is the probability that the passenger will ride in each of the 3 cars?

(A) 0      (B)  $1/9$       (C)  $2/9$       (D)  $1/3$       (E) 1

解析：第一次的概率： $3/3$ ；第二次的概率： $2/3$ ；第三次的概率： $1/3$   
概率  $3/3 * 2/3 * 1/3 = 2/9$

(总的可能性： $3*3*3=27$ ， 符合条件的可能性： $3*2*1=6$ )

答案：C

例题 7 (GWD T1Q37)

A jar contains 16 marbles, of which 4 are red, 3 are blue, and the rest are yellow. If 2 marbles are to be selected at random from the jar, one at a time without being replaced, what is the probability that the first marble selected will be red and the second marble selected will be blue?

(A)  $3/64$       (B)  $1/20$       (C)  $1/16$       (D)  $1/12$       (E)  $1/8$

解析：第一个的概率： $4/16$ ；第二个的概率： $3/15$

概率  $4/16 * 3/15 = 1/20$

(总的可能性： $C_{16}^1 * C_{15}^1=240$ ； 符合条件的可能性： $C_4^1 * C_3^1 =4*3=12$ )

答案：B

例题 8 (GWD T1Q9)

A gardener is going to plant 2 red rosebushes and 2 white rosebushes. If the gardener is to select each of the bushes at random, one at a time, and plant them in a row, what is the probability that the 2 rosebushes in the middle of the row will be the red rosebushes?

(A)  $1/12$       (B)  $1/6$       (C)  $1/5$       (D)  $1/3$       (E)  $1/2$

解析：第一个的概率： $2/4$ ；第二个的概率： $2/3$ ；第三个的概率： $1/2$ ；第四个

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的概率:  $1/1$

概率  $2/4 * 2/3 * 1/2 * 1/1 = 4/24 = 1/6$

(总的可能性:  $P_4^4=24$ , 符合条件的可能性:  $2*2*1*1=4$ )

答案: B

例题 9 (GWD T9Q23)

When tossed, a certain coin has equal probability of landing on either side. If the coin is tossed 3 times, what is the probability that it will land on the same side each time?

- (A)  $\frac{1}{8}$       (B)  $\frac{1}{4}$       (C)  $\frac{1}{3}$       (D)  $\frac{3}{8}$       (E)  $\frac{1}{2}$

解析: (1) 3次同为正面:  $1/2 * 1/2 * 1/2 = 1/8$

(2) 3次同为反面:  $1/2 * 1/2 * 1/2 = 1/8$

总的概率:  $1/4$

答案: B

例题 10 (GWD T15Q22)

How many 4-digit positive integers are there in which all 4 digits are even?

- (A) 625      (B) 600      (C) 500      (D) 400      (E) 256

解析: 第一个数的可能性: 4种 (不能为0),

第二个数的可能性: 5种,

第三个数的可能性: 5种,

第四个数的可能性: 5种,

总共的可能性:  $4*5*5*5 = 500$

答案: C

例题 11:

A three-digit code for certain logs uses the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 according to the following constraints. The first digit cannot be 0 or 1, the second digit must be 0 or 1, and the second and third digits cannot both be 0 in the same code. How many different codes are possible?

- A. 144
- B. 152
- C. 160
- D. 168
- E. 176

解析：第一位的可能性：8；  
第二位的可能性：2；  
第三位的可能性：10/9 种（取决于第二位的数值是否为 0）  
 $8*1*10$ （第二位为 1） +  $8*1*9$ （第二位为 0） = 152

答案：B

#### 例题 12 (GWD T25Q14)

If a certain coin is flipped, the probability that the coin will land heads up is  $\frac{1}{2}$ . If the coin is flipped 5 times, what is the probability that it will land heads up on the first 3 flips and not on the last 2 flips?

- A.  $\frac{3}{5}$
- B.  $\frac{1}{2}$
- C.  $\frac{1}{5}$
- D.  $\frac{1}{8}$
- E.  $\frac{1}{32}$

解析： $\frac{1}{2} * \frac{1}{2} * \frac{1}{2} * \frac{1}{2} * \frac{1}{2} = \frac{1}{32}$

答案：E

#### 考查形式 4：正难则反

正常解决非常麻烦，则反过来思考，非常类似于逻辑假设题的“取反”。

题目要求 A 事件发生的可能性，我们先求总的可能性，然后求出“非

A”的可能性。**A 的可能性 = 总的可能性 - 非 A 的可能性。**



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一般情况下，题目中如果说“不能……”，“at least……”，基本都可以用正难则反来解决。

例题 1:

A certain law firm consists of 4 senior partners and 6 junior partners. How many different groups of 3 partners can be formed in which at least one member of the group is a senior partner? (Two groups are considered different if at least one group member is different.)

- (A) 48      (B) 100      (C) 120      (D) 288      (E) 600

解析:  $C_{10}^3 - C_6^3$

答案: B

例题 2 (GWD T2Q26)

There are 8 books on a shelf, of which 2 are paperbacks and 6 are hardbacks. How many possible selections of 4 books from this self include at least one paperback?

- (A) 40      (B) 45      (C) 50      (D) 55      (E) 60

解析: 总的可能性:  $C_8^4 = 70$ ,

抽到的 4 本书没有一本是 paperback (全都是 hardback) 的可能性:  $C_6^4 = 15$ .

所以抽到的 4 本书中至少有一本是 paperback 的可能性是:  $70 - 15 = 55$ .

答案: D

例题 3 (GWD T2Q31)

There are 8 magazines lying on a table; 4 are fashion magazines and the other 4 are sports magazines. If 3 magazines are to be selected at random from the 8 magazines, what is the probability that at least one of the fashion magazines will be selected?

- (A) 1/2      (B) 2/3      (C) 32/35      (D) 11/12      (E) 13/14

解析: 8 本书中抽 3 本的可能性:  $C_8^3 = 56$

抽到的 3 本书没有一本是 fashion magazine (全是 sport magazine) 的可能性:  
 $C_4^3 = 4$

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抽到的3本书至少有一本 fashion magazine 的可能性： $56 - 4 = 52$

概率是： $52/56 = 13/14$

答案：E

例题4 (GWD T24Q4)

A shipment of 8 television sets contains 2 black-and-white sets and 6 color sets. If 2 television sets are to be chosen at random from this shipment, what is the probability that at least 1 of the 2 sets chosen will be a black-and-white set?

- A.  $1/7$    B.  $1/4$    C.  $5/14$    D.  $11/28$    E.  $13/26$

解析：总的可能性： $C_8^2 = 28$

一台黑白电视机也没抽到的可能性（两次都是彩色电视）： $C_6^2 = 15$

抽到至少一台黑白电视机的可能性： $C_8^2 - C_6^2 = 13$

概率是： $13/28$

答案：E

例题5 (GWD T26Q3)

A coin that is tossed will land heads or tails, and each outcome has equal probability. What is the probability that the coin will land heads at least once on two tosses?

- A.  $1/4$   
B.  $1/3$   
C.  $1/2$   
D.  $2/3$   
E.  $3/4$

解析：总的可能性： $2 * 2 = 4$

没抛出一枚正面的可能性（两次都是反面）： $1 * 1 = 1$

抛出至少一枚正面的可能性： $4 - 1 = 3$

概率是： $3/4$

答案：E

例题 6: (4 月机经)

医生有 7 种可以治疗某种病的药: ABCDEFG, 其中 AB 不能一起用, 否则会无效。  
问选三种药的组合且有效的有多少种?

思路:

总的可能性: 七种药里面选三种  $C_7^3 = 35$

与条件相反的可能性“偏偏 AB 一起用”:  $C_2^2 * C_5^1 = 5$

则有效为  $35 - 5 = 30$

答案: 30

例题 7:

If a committee of 3 people is to be selected from among 5 married couples so that the committee does not include two people who are married to each other, how many such committees are possible?

(A) 20 (B) 40 (C) 50 (D) 80 (E) 120

解析: 从 10 个人里抽 3 个人总的可能性:  $C_{10}^3 = 120$

抽到的人包含一对夫妻的可能性:  $C_5^1$  (5 对夫妻中抽中 1 对) \*  $C_8^2$  (剩下 8 个人中随机抽一个) = 40

所以抽到的人不能包含一对夫妻的可能性 =  $120 - 40 = 80$

答案: D

例题 8 (11 月机经):

正方体选三条边涂色, 涂色的这三条边不能相交于一点, 有几种涂法?

解析:

总的可能性:  $C_{12}^3 = 220$

涂色的三条边刚好相交于一点的可能性 (总共 8 个交点): 8 种

三条边并不相交于一点的可能性:  $220 - 8 = 212$

考查形式 5: 捆绑

做法:

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要求某几个元素必须排在一起，可以先把这些必须排在一起的元素看作是一个元素，和其他元素进行排列，然后对这些必须排在一起的元素再进行内部排序。

**举例：**

ABCDE 五个元素进行排列，其中 BC 必须排在一起，则先把 B 和 C 看作是一同元素，即  $P_4^4$ ，然后 B 和 C 内部排序是  $P_2^2$ ，所以总的可能性是  $P_4^4 * P_2^2$

**适用情况：**

A 和 B 必须在一起（挨着）、A 和 B 不能在一起（中间是隔开的）

典型例题：

There will be 5 songs and 3 dances in a performance. How many distinguished way to arrange the shows if all dances cannot be next to each other?

- (1) How many distinguished way to arrange the shows?
- (2) How many distinguished way to arrange the shows if all dances must be next to each other?
- (3) How many distinguished way to arrange the shows if all dances cannot be next to each other?

答案：

- (1)  $P_8^8$
- (2)  $P_6^6 * P_3^3$
- (3)  $P_8^8 - P_6^6 * P_3^3$

例题 1：

一个幼儿园有 three pairs of twins，让他们站成一排照相，每一对双胞胎都站在一起。请问一共有几种排法？

**思路：**

三对双胞胎，每一对是一个整体，三个整体排序就是  $P_3^3=6$   
每一对整体分成两个人，Aa, Bb, Cc，每对双胞胎内部排序： $P_2^2$ 。

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所以结果是  $P_3^3 \times P_2^2 \times P_2^2 \times P_2^2 = 48$

答案： 48

例题 2（5 月机经）：

班里有 20 个人 名字按照首字母顺序排，老师选三个人，不是连续着的三个人的选法有多少？

思路：

用 20 个连续的字母分别代表一个学生，老师每次从中选 3 人，但是不会选择连续的 3 个字母。  $C_{20}^3 - 18 = 1122$

答案： 1122

例题 3：

ABCDE 五个人，五个人排序，AB 之间至少要隔一个人，问有多少排序方法？

思路：

排列总数为  $P_5^5 = 120$

先算出 AB 排在一起的可能性：将 AB 捆绑为一个元素，即对 4 个元素进行排列；因为 AB 是两个人，还需要对这个捆绑的元素进行内部排序。

$$P_4^4 * P_2^2 = 48$$

AB 不能排在一起的可能性：  $120 - 48 = 72$

答案： 72

例题 4：（4 月机经）

C I R C L E 六个字母可以组成一个 strain，问两个 C 之间有其他字母隔开的 strain 有多少种？

思路：

总的可能性：  $\frac{P_6^6}{2!} = 360$

与条件相反的可能性“两个 C 必须在一起”：  $P_5^5 = 120$ （因为两个 C 是重复元素，

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不需要再对两个 C 进行内部排序)  
那么两个 C 之间有其他字母隔开的可能性:  $360 - 120 = 240$ 。

答案: 240

### 考查形式 6: 逆推

做法: 题目中告诉你已知的可能性, 将已知元素个数代入公式, 求未知的元素个数。

例题 1:

To furnish a room in a model home, an interior decorator is to select 2 chairs and 2 tables from a collection of chairs and tables in a warehouse that are all different from each other. If there are 5 chairs in the warehouse and if 150 different combinations are possible, how many tables are in the warehouse?

(A) 6      (B) 8      (C) 10      (D) 15      (E) 30

答案: A

例题 2:

In a stack of cards, 9 cards are blue and the rest are red. If 2 cards are to be chosen at random from the stack without replacement, the probability that the cards chosen will both be blue is  $\frac{6}{11}$ . What is the number of cards in the stack?

- A. 10
- B. 11
- C. 12
- D. 15
- E. 18

解析: 设红球有  $n$  个。

$$\text{总的可能性: } C_{9+n}^2 = \frac{(n+9) \cdot (n+8)}{2}$$

$$\text{抽到的 2 个球都是篮球的可能性: } C_9^2 \cdot C_n^0 = 36$$

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因为概率是  $\frac{6}{11}$ ，所以  $C_{9+n}^2 = \frac{(n+9) * (n+8)}{2} = 66$

解出  $n = 3$ 。 所以总数： $n+9=12$

答案： C

例题 3（5 月机经）：

The company is to select 3 of  $n$  of their employees to participate in an exhibition show. How many of different groups of the 3 employees can be formed?

(1) If the company is to select 2 employees as a group, 105 different groups can be formed.

(2) If the company has  $n+1$  employees, there will be 105 more groups can be formed compared to current one

解题思路：

所以是要求  $C(n,3)$

(1)  $C(n,2)=105$ , 那么  $n(n-1)/2 = 105 \dots n=15$  那么也就可以求出  $C(n,3)$  了。

(2)  $C(n+1,3)-C(n,3)=105$ ;  $[(n+1)*n*(n-1)] / (3*2*1) - [n*(n-1)*(n-2)] / (3*2*1) = 105$  ,  $n=15$  也可以知道  $C(n,3)$ 。

答案： D

例题 4（GWD T12Q6）：

A box contains 10 light bulbs, fewer than half of which are defective. Two bulbs are to be drawn simultaneously from the box. If  $n$  of the bulbs in box are defective, what is the value of  $n$ ?

(1) The probability that the two bulbs to be drawn will be defective is  $\frac{1}{15}$ .

(2) The probability that one of the bulbs to be drawn will be defective and the other will not be defective is  $\frac{7}{15}$ .

解析：

(1) 10 个灯泡抽 2 个的可能性： $C_{10}^2 = 45$

抽到的 2 个灯泡都是 defective 的可能性： $C_n^2 = n(n-1)/2$

概率是  $\frac{1}{15}$ ，则  $C_n^2 = n(n-1)/2 = 3$ ， $n$  为正整数， $n=3$ 。

(2) 10 个灯泡抽 2 个的可能性： $C_{10}^2 = 45$

抽到的 2 个灯泡一个坏、一个不坏的可能性： $C_n^1 * C_{10-n}^1 = n*(10-n)$

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概率是  $\frac{7}{15}$ , 则  $C_n^1 * C_{10-n}^1 = n*(10-n) = 21$   
 $n=3$  或  $7$ , 因为题干中已经说明  $n < 5$ , 所以  $n=3$ .

答案: D

例题 5 (GWD T14Q2):

If 2 different representatives are to be selected at random from a group of 10 employees and if  $p$  is the probability that both representatives selected will be

women, is  $p > \frac{1}{2}$ ?

(1) More than  $\frac{1}{2}$  of the 10 employees are women.

(2) The probability that both representatives selected will be men is less than  $\frac{1}{10}$ .

解析: 设女性有  $x$  名, 则男性有  $10-x$  名。

10 个人里抽 2 个人的可能性:  $C_{10}^2 = 45$

抽到的 2 个人都是女性的可能性:  $C_x^2 = \frac{x(x-1)}{2}$

概率是:  $\frac{x(x-1)}{90}$ 。

题干:  $\frac{x(x-1)}{90} > \frac{1}{2}$ , 即  $x^2-x > 45$ . 即判断  $x^2-x$  和 45 的大小

(1)  $x > 5$   $x=6$  或  $7$  时,  $x^2-x < 45$ ;  $x=8$  或  $9$  时,  $x^2-x > 45$ . insufficient

(2) 10 个人里抽 2 个人的可能性:  $C_{10}^2 = 45$

抽到的 2 个人都是男性的可能性:  $C_{10-x}^2 = \frac{(10-x)(9-x)}{2}$

概率是:  $\frac{(10-x)(9-x)}{90}$ 。  $\frac{(10-x)(9-x)}{90} < \frac{1}{10}$ , 即  $(10-x)(9-x) < 9$

代入  $x$  ( $1 \sim 8$ ),  $x=7$  或  $8$  时,  $(10-x)(9-x) < 9$ , 符合条件。

$x=7$  时,  $x^2-x < 45$ ;  $x=8$  时,  $x^2-x > 45$ . insufficient

(1) + (2):  $x=7$  或  $8$ . insufficient

答案: E

例题 6 (4 月机经):

A,B,C 三个人要先后从 X 和 Y 罐中拿球出来: X 罐里面有 3 个红球, 10 个白球; Y 罐里面有 2 个红球, 8 个白球。A 和 B 两个人都先分别拿了一个球了, 问 C 这个人应该拿哪个罐子里面的球才能使拿到红球的概率更大一些?

(1) A 是从 X 罐里面拿了一个红球

(2) B 从 Y 罐中拿了一个红球



思路:

四种情况: A 从 X 罐拿一个红球 B 从 X 罐拿一个红球;

A 从 X 罐拿一个红球 B 从 Y 罐拿一个红球;

A 从 Y 罐拿一个红球 B 从 X 罐拿一个红球;

A 从 Y 罐拿一个红球 B 从 Y 罐拿一个红球。

(1) 如果是 A 从 X 罐拿一个红球 B 从 X 罐拿一个红球, X 罐取到红球的概率是  $1/11$ , Y 罐取红球的概率是  $2/10$ , Y 罐概率更大;

如果是 A 从 X 罐拿一个红球 B 从 Y 罐拿一个红球, X 罐取到红球的概率是  $2/12$ , Y 罐取红球的概率是  $1/9$ , X 罐概率更大。

所以不确定

(2) A 从 X 罐拿一个红球 B 从 Y 罐拿一个红球, X 罐取到红球的概率是  $2/12$ , Y 罐取红球的概率是  $1/9$ , X 罐概率更大;

如果是 A 从 Y 罐拿一个红球 B 从 Y 罐拿一个红球, X 罐取到红球的概率是  $3/13$ , Y 罐取红球的概率是 0, X 罐概率更大。

所以 X 罐取到红球的概率更大。

答案: B

### 考查形式 7: 重复元素

公式: 
$$\frac{A_n^n}{(\text{元素1的个数!}) * (\text{元素2的个数!})}$$

例题 1:

In how many distinguishable ways can the 7 letters in the word MINIMUM be arranged, if all the letters are used each time?

- (A) 7      (B) 42      (C) 420      (D) 840      (E) 5040

答案: C

例题 2:

There are 5 cars to be displayed in 5 parking spaces with all the cars facing the same direction. Of the 5 cars, 3 are red, 1 is blue, and 1 is yellow. If the cars are identical except for color, how many different display arrangements of the 5 cars are possible?

(A) 20      (B) 25      (C) 40      (D) 60      (E) 125

答案: A

例题 3:

How many different 6-letter sequences are there that consist of 1 A, 2 B's, and 3 C's ?

- A. 6
- B. 60
- C. 120
- D. 360
- E. 720

解析:  $\frac{P_6^6}{2! * 3!}$

答案: B

例题 4:

一个公司买了 5 台电脑, 2 台是 A 品牌, 3 台是 B 品牌, 分给七个办公室, 每个办公室最多有一台, 问有多少种分法?

思路:

第一台电脑可以有 7 种分法, 第二种 6 种。电脑有重复

$$\frac{7*6*5*4*3}{3!*2!} = 210$$

答案: 210

**考查形式 8: 圆桌排座**

公式:  $\frac{A_n^n}{n} = \frac{n!}{n} = (n-1)!$

例题 1:

At a dinner party, 5 people are to be seated around a circular table. Two seating

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arrangements are considered different only when the positions of the people are different relative to each other. What is the total number of different possible seating arrangements for the group?

- (A) 5      (B) 10      (C) 24      (D) 32      (E) 120

答案: C

GMAT 数学