

2018-2019 学年高三第一次模拟试题参考答案 数学 (理科)

一. 选择题: 1. A 2. C 3. D 4. B 5. A 6. D 7. D 8. A 9. B 10. B 11. A 12. C

二. 填空题: 13. $y = -\frac{1}{4}$ 14. 15 15. $\sqrt{22}$ 16. 1

三. 解答题:

17. 解 (I) $Q a \sin A + (c - a) \sin C = b \sin B$, 由 $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ 得 $a^2 + c^2 - ac = b^2$,

由余弦定理得 $\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{1}{2}$,4 分

$Q 0 < B < \pi, \therefore B = 60^\circ$;6 分

(II) 连接 CE , $Q D$ 是 AC 的中点, $DE \perp AC, \therefore AE = CE$,

$\therefore CE = AE = \frac{DE}{\sin A} = \frac{\sqrt{6}}{2 \sin A}$,7 分

在 $\triangle BCE$ 中, 由正弦定理得 $\frac{CE}{\sin B} = \frac{BC}{\sin \angle BEC} = \frac{BC}{\sin 2A}$,

$\therefore \frac{\sqrt{6}}{2 \sin A \sin 60^\circ} = \frac{2}{2 \sin A \cos A}, \therefore \cos A = \frac{\sqrt{2}}{2}$,

$Q 0 < A < \pi, \therefore A = 45^\circ$,9 分

$\therefore \angle ACB = 75^\circ, \therefore \angle BCE = \angle ACB - \angle ACE = 30^\circ, \angle BEC = 90^\circ$,

$\therefore CE = AE = \sqrt{3}, AB = AE + BE = \sqrt{3} + 1$,

$\therefore S_{\triangle ABC} = \frac{1}{2} AB \cdot CE = \frac{3 + \sqrt{3}}{2}$12 分

18. (I) 证明: $Q CDEF$ 是菱形, $\therefore DE = CD = 2AD, CE \perp DF$,1 分

$Q AE = \sqrt{5}AD, \therefore AE^2 = AD^2 + DE^2 = 5AD^2, \therefore AD \perp DE$,

$Q AD \perp CD, \therefore AD \perp \text{面 } CDEF, \therefore AD \perp CE$,4 分

$\therefore CE \perp \text{面 } ADF, \therefore CE \perp AF$;6 分

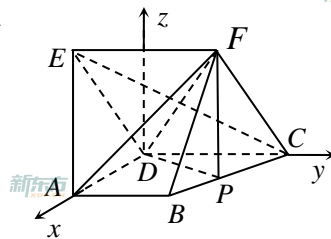
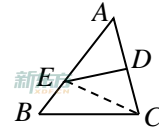
(II) 由 (I) 知以 D 为坐标原点, \overrightarrow{DA} 的方向为 x 轴的正方向, $|\overrightarrow{DA}|$ 为单位长, 建立如图的空间直角坐标系 $D-xyz$,

由题设可得 $A(1, 0, 0), B(1, 1, 0), C(0, 2, 0), E(0, -1, \sqrt{3}),$

$F(0, 1, \sqrt{3}), \therefore \overrightarrow{CP} = \lambda \overrightarrow{CB} = (\lambda, -\lambda, 0)$

$\therefore \overrightarrow{DP} = \overrightarrow{DC} + \overrightarrow{CP} = (\lambda, 2 - \lambda, 0),$

设 $\vec{m} = (x, y, z)$ 是平面 DFP 的一个法向量, 则 $\begin{cases} \vec{m} \cdot \overrightarrow{DF} = 0, \\ \vec{m} \cdot \overrightarrow{DP} = 0, \end{cases} \therefore \begin{cases} y + \sqrt{3}z = 0, \\ \lambda x + (2 - \lambda)y = 0, \end{cases}$



令 $z = -1$, 则
$$\begin{cases} y = \sqrt{3}, \\ x = \sqrt{3}(1 - \frac{2}{\lambda}), \end{cases} \therefore \vec{m} = (\sqrt{3}(1 - \frac{2}{\lambda}), \sqrt{3}, -1), \dots\dots\dots 9 \text{分}$$

由 (I) 可知 $\vec{CE} = (0, -3, \sqrt{3})$ 是平面 ADF 的一个法向量,

Q 二面角 $A-DF-P$ 的大小为 60° ,

$$\therefore \cos 60^\circ = \frac{|\vec{m} \cdot \vec{CE}|}{|\vec{m}| \cdot |\vec{CE}|} = \frac{|-4\sqrt{3}|}{2\sqrt{3} \cdot \sqrt{3(-\frac{2}{\lambda})^2 + 4}} = \frac{1}{2}, \therefore \lambda = \frac{2}{3}. \dots\dots\dots 12 \text{分}$$

19. 解: (I) 由题意得 $z = \ln y = \ln e^{bx+a} = bx + a$,

$$\therefore \bar{z} = \frac{\sum_{i=1}^7 x_i z_i - 7\bar{x}\bar{z}}{\sum_{i=1}^7 x_i^2 - 7\bar{x}^2} = \frac{112 - 7 \times 4 \times 3.5}{140 - 7 \times 4^2} = 0.5, \therefore \bar{a} = \bar{z} - \bar{b}\bar{x} = 3.5 - 0.5 \times 4 = 1.5,$$

$\therefore z$ 关于 x 的线性回归方程为 $z = 0.5x + 1.5$, $\dots\dots\dots 3 \text{分}$

$\therefore y$ 关于 x 的回归方程为 $y = e^{0.5x+1.5}$, 当 $x = 8$ 时, $y = e^{5.5} = 244.69$,

\therefore 第 8 天使用扫码支付的人次为 2447; $\dots\dots\dots 6 \text{分}$

(II) 由题意得 ξ 的所有取值为 0.5, 0.7, 0.9, 1,

$P(\xi = 0.5) = \frac{1}{3} \times 30\% = 0.10, P(\xi = 0.7) = 60\% + \frac{1}{2} \times 30\% = 0.75,$

$P(\xi = 0.9) = \frac{1}{6} \times 30\% = 0.05, P(\xi = 1) = 10\% = 0.10,$

$\therefore \xi$ 的分布列为:

ξ	0.5	0.7	0.9	1
P	0.10	0.75	0.05	0.10

$\dots\dots\dots 10 \text{分}$

$\therefore E\xi = 0.5 \times 0.10 + 0.7 \times 0.75 + 0.9 \times 0.05 + 1 \times 0.10 = 0.72. \dots\dots\dots 12 \text{分}$

20. 解: (I) 由题意得
$$\begin{cases} 2a + 2c = 6, \\ \frac{1}{2} \times 2bc = \sqrt{3}, \dots\dots\dots 3 \text{分} \\ a^2 = b^2 + c^2, \end{cases}$$

$$\therefore \begin{cases} c = 1, \\ b = \sqrt{3}, \\ a = 2, \end{cases} \therefore \text{椭圆 } C \text{ 的方程为 } \frac{x^2}{4} + \frac{y^2}{3} = 1; \dots\dots\dots 6 \text{分}$$

(II) 由 (I) 得 $A(-2, 0), B(2, 0), F_2(1, 0)$, 设直线 MN 的方程为 $x = my + 1$,

$$M(x_1, y_1), N(x_2, y_2), \text{ 由 } \begin{cases} x = mx + 1, \\ \frac{x^2}{4} + \frac{y^2}{3} = 1 \end{cases} \text{ 得 } (4 + 3m^2)y^2 + 6my - 9 = 0,$$

$$\therefore y_1 + y_2 = -\frac{6m}{4 + 3m^2}, \quad y_1 y_2 = -\frac{9}{4 + 3m^2}, \quad \therefore m y_1 y_2 = \frac{3}{2}(y_1 + y_2), \dots\dots\dots 9 \text{ 分}$$

Q 直线 AM 的方程为 $y = \frac{y_1}{x_1 + 2}(x + 2)$, 直线 BN 的方程为 $y = \frac{y_2}{x_2 - 2}(x - 2)$,

$$\therefore \frac{y_1}{x_1 + 2}(x + 2) = \frac{y_2}{x_2 - 2}(x - 2), \quad \therefore \frac{x + 2}{x - 2} = \frac{y_2(x_1 + 2)}{y_1(x_2 - 2)} = \frac{m y_1 y_2 + 3 y_2}{m y_1 y_2 - y_1} = 3,$$

$\therefore x = 4$, \therefore 直线 AM 与 BN 的交点在直线 $x = 4$ 上. $\dots\dots\dots 12 \text{ 分}$

21.(I)解: 由题意得 $f'(x) = \frac{1}{x} - 2ax + (2 - a) = -\frac{(2x + 1)(ax - 1)}{x}, \quad x > 0, \dots\dots\dots 2 \text{ 分}$

(1) 当 $a \leq 0$ 时, $f'(x) > 0$ 在 $(0, +\infty)$ 上恒成立, $\therefore f(x)$ 在 $(0, +\infty)$ 上单调递增;

(2) 当 $a > 0$ 时, 令 $f'(x) > 0$ 则 $0 < x < \frac{1}{a}$; 令 $f'(x) < 0$ 则 $x > \frac{1}{a}$,

$\therefore f(x)$ 在 $(0, \frac{1}{a})$ 上单调递增, 在 $(\frac{1}{a}, +\infty)$ 上单调递减; $\dots\dots\dots 6 \text{ 分}$

(II)证明: 当 $a < -\frac{1}{2}$ 时, $Q \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{1}{x_2 - x_1} \ln \frac{x_2}{x_1} - a(x_2 + x_1) + (2 - a),$

$$f'(x_0) = \frac{1}{x_0} - 2ax_0 + (2 - a)$$

$$\therefore \frac{1}{x_2 - x_1} \ln \frac{x_2}{x_1} - a(x_2 + x_1) = \frac{1}{x_0} - 2ax_0, \dots\dots\dots 7 \text{ 分}$$

$$Q f'(\frac{x_1 + x_2}{2}) - f'(x_0) = \frac{2}{x_2 + x_1} - a(x_2 + x_1) - (\frac{1}{x_0} - 2ax_0) = \frac{2}{x_2 + x_1} - \frac{1}{x_2 - x_1} \ln \frac{x_2}{x_1}$$

$$= \frac{1}{x_2 - x_1} \left[\frac{2(x_2 - x_1)}{x_2 + x_1} - \ln \frac{x_2}{x_1} \right] = \frac{1}{x_2 - x_1} \left[\frac{2(\frac{x_2}{x_1} - 1)}{\frac{x_2}{x_1} + 1} - \ln \frac{x_2}{x_1} \right], \dots\dots\dots 9 \text{ 分}$$

令 $t = \frac{x_2}{x_1}, \quad g(t) = \frac{2(t - 1)}{t + 1} - \ln t, \quad t > 1$, 则 $g'(t) = -\frac{(t - 1)^2}{t(t + 1)^2} < 0, \quad \therefore g(t) < g(1) = 0,$

$$\therefore f'(\frac{x_1 + x_2}{2}) - f'(x_0) < 0, \quad \therefore f'(\frac{x_1 + x_2}{2}) < f'(x_0), \dots\dots\dots 11 \text{ 分}$$

设 $h(x) = f'(x) = \frac{1}{x} - 2ax + (2 - a), \quad x > 1$, 则 $h'(x) = -\frac{1}{x^2} - 2a > -1 + 1 = 0,$

$$\therefore h(x) = f'(x) \text{ 在 } (1, +\infty) \text{ 上单调递增, } \therefore \frac{x_1 + x_2}{2} < x_0. \dots\dots\dots 12 \text{ 分}$$

22 解: (I) $Q \rho = 2 \cos \theta, \therefore$ 曲线 C_2 的直角坐标方程为 $\therefore (x - 1)^2 + y^2 = 1,$

Q α 是曲线 $C_1: \begin{cases} x = t \cos \alpha, \\ y = 1 + t \sin \alpha \end{cases}$ 的参数, $\therefore C_1$ 的普通方程为 $x^2 + (y-1)^2 = t^2$,2 分

Q C_1 与 C_2 有且只有一个公共点, $\therefore |t| = \sqrt{2} - 1$ 或 $|t| = \sqrt{2} + 1$,

$\therefore C_1$ 的普通方程为 $x^2 + (y-1)^2 = (\sqrt{2}-1)^2$ 或 $x^2 + (y-1)^2 = (\sqrt{2}+1)^2$;5 分

(II) Q t 是曲线 $C_1: \begin{cases} x = t \cos \alpha, \\ y = 1 + t \sin \alpha \end{cases}$ 的参数, $\therefore C_1$ 是过点 $A(0,1)$ 的一条直线,6 分

设与点 P, Q 相对应的参数分别是 t_1, t_2 , 把 $\begin{cases} x = t \cos \alpha, \\ y = 1 + t \sin \alpha \end{cases}$ 代入 $(x-1)^2 + y^2 = 1$ 得

$$t^2 + 2(\sin \alpha - \cos \alpha)t + 1 = 0, \therefore \begin{cases} t_1 + t_2 = -2\sqrt{2} \sin(\alpha - \frac{\pi}{4}), \dots\dots\dots 8 \text{ 分} \\ t_1 \cdot t_2 = 1, \end{cases}$$

$$\therefore \frac{1}{|AP|} + \frac{1}{|AQ|} = \frac{1}{|t_1|} + \frac{1}{|t_2|} = |t_1| + |t_2| = |t_1 + t_2| = 2\sqrt{2} |\sin(\alpha - \frac{\pi}{4})| \leq 2\sqrt{2},$$

当 $\alpha = \frac{3\pi}{4}$ 时, $\Delta = 4(\sin \alpha - \cos \alpha)^2 - 4 = 4 > 0$,

$\frac{1}{|AP|} + \frac{1}{|AQ|}$ 取最大值 $2\sqrt{2}$10 分

23 (I) 解: Q $f(x) = |2x-1| + 2|x+1| \leq 5, \therefore |x - \frac{1}{2}| + |x+1| \leq \frac{5}{2}$,

由绝对值得几何意义可得 $x = -\frac{3}{2}$ 和 $x = 1$ 上述不等式中的等号成立,3 分

\therefore 不等式 $f(x) \leq 5$ 的解集为 $[-\frac{3}{2}, 1]$;5 分

(II) 由绝对值得几何意义易得 $f(x) = 2(|x - \frac{1}{2}| + |x+1|)$ 的最小值为 3,

$\therefore 3 \leq 5 + m - m^2, \therefore -1 \leq m \leq 2, \therefore M = 2, \therefore a^3 + b^3 = 2, \dots\dots\dots 7 \text{ 分}$

Q $2 = a^3 + b^3 = (a+b)(a^2 - ab + b^2), a^2 - ab + b^2 \geq 0, \therefore a+b > 0, \dots\dots\dots 8 \text{ 分}$

$$Q 2ab \leq a^2 + b^2, \therefore 4ab \leq (a+b)^2, \therefore ab \leq \frac{(a+b)^2}{4},$$

$$Q 2 = a^3 + b^3 = (a+b)(a^2 - ab + b^2) = (a+b)[(a+b)^2 - 3ab] \geq \frac{1}{4}(a+b)^3, \therefore a+b \leq 2,$$

$\therefore 0 < a+b \leq 2. \dots\dots\dots 10 \text{ 分}$